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Financial Weather Derivatives for Corn Production in Northeastern China: Modelling the Underlying Weather Index

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Financial Weather Derivatives for Corn Production in Northeastern China: Modelling the underlying Weather Index

Abstract

The focus in this study is on estimating the underlying weather index for pricing financial derivatives to hedge weather risks in crop production. Different index estimation methods for growing degree days (GDDs) are compared. In particular, daily average temperatures for deriving GDDs are simulated using an econometric model and a stochastic process that uses three methods to estimate the mean-reversion parameter. Finally, the historical approach based on a five-year moving average of GDDs is compared with the econometric and stochastic models. Results indicate that econometric model provides the best fit, followed by the the historical average method and then the stochastic process with a high mean reversion parameter. Premiums from the econometric model with sine function and historical average approaches are closer to those based on realized weather values compared with the stochastic approach. Therefore, the econometric model with sine function and the historical average approach provide better pricing estimates than other methods.

Key Words: Pricing weather options; weather-based derivatives; stochastic process and econometric modeling; growing degree days; agricultural finance

JEL Categories: Q14, G11, G12, G32

1. Introduction

Traditional crop insurance is used to protect against losses of crop yields caused primarily by adverse weather. A payout occurs when actual yield is below a predetermined reference level. However, adverse selection arises when only those who are most likely to claim benefits join the program, at the expense of higher premiums and lower uptake. Also, such a program has the problem of moral hazard, which happens because insured farmers take no measures to reduce their potential risks. Financial weather derivatives and weather-indexed insurance are alternative financial instruments that can be used to hedge production risks related to weather outcomes. Payoffs depend on the outcome of a weather index that represents the weather conditions. The problem of moral hazard disappears since the value of the weather index does not depend on individuals' actions. Although weather derivatives and weather indexed insurance are essentially similar, there exist mature exchange markets for financial weather derivatives while weather-indexed insurance relies only on over-the-counter (OTC) contracts. Another important difference is that financial weather derivatives not only provide economic agents with a tool for hedging weather risks, but also provide an investment instrument that participants can purchase for diversifying their investment portfolios.

Financial weather derivatives consist of future contracts, options and swaps. A call option can be claimed when the value of the weather index is above a specified threshold value, while a put option can be claimed when the outcome of the weather index is below a specified threshold. The cost of acquiring an option is its premium. In the agriculture, weather derivatives can be used to protect against crop losses caused by cold weather, extreme heat or too much or too little rainfall. For example, a farmer could hedge against too little warmth, as measured by growing degree days (GDDs), or too little cumulative rainfall (CR), by purchasing a put option that reduces the financial risk that these weather variables adversely affect crop yields. If the weather outcomes below the threshold value, a payout would be claimed; if they are at or above the threshold value, the farmer would not claim the option and lose the premium paid for the option contract. In that case, yields are likely higher than expected, which would compensate for the premium.

A number of studies have focused on methods for pricing weather derivative contracts, including Alaton et al. (2002), Brody et al. (2002), Campbell and Diebold (2005), and Jewson et al. (2005). In these studies, parametric or non-parametric methods were used to specify a probability distribution of the weather index, employing historical average method or stochastic processes to model weather outcomes. Most studies of weather derivatives focused on market-based, market-traded heating degree day (HDD) or cooling degree day (CDD) indexes in the energy sector (Huang et al. 2008; Goncu 2011; Schiller et al. 2012). In agriculture, where financial weather derivatives have not been adopted on the same scale as in the energy sector, studies have looked at rainfall or heat index-based weather derivatives, using historical data to construct such indexes (Turvey 2001; Stoppa & Hess 2003; Vedenov & Barnett 2004; Musshoff et al. 2011; Sun & Lou 2013).

The main objective of the current study is to examine index estimation methods for the pricing of weather derivatives, and compare methods on the basis of historic weather conditions and weather predictions. The methods used to price weather derivatives based on GDDs are a weather index distribution method using historic averages (burn analysis), an estimated non-stochastic sine function, and a stochastic process with three approaches for estimating its mean-reverting parameter.

The application is to a major corn growing region in northeastern China, namely Heilongjiang Province. Crop yields in northern China are highly dependent on growing season weather conditions, especially heat conditions during the growing season (Sun & van Kooten 2014; Chen et al. 2011). Therefore, farmers could use a GDD-based financial weather product to mitigate weather risk. China is the second largest maize producing country in the world after the United States (FAO 2010), with corn production in Northeastern China (Heilongjiang, Jilin and Liaoning provinces) accounting for more than 30% of the country's total corn production in 2010 (China Statistical Yearbook 2011). Weather index insurance was introduced into China in 2008 (Liu et al. 2010), but it was only adopted in some pilot areas.

2. Data Analysis

Heilongjiang Province is located in the most Northeastern part of China, covering an area of 47.3 million hectares (M ha), and lying 121º11′-135º43′ E and 43º25′-53º33′ N, and 50-200 meters above sea level (asl).[1] It is part of China's main spring corn production area. The daily average temperature data over the period 1985 to 2015 (31 years with 8 leap years) constitute 11,322 daily observations and are from the China Meteorological Data Sharing System. Plots of daily average temperatures for the decade 2006 to 2015 from a weather station in Heilongjiang Province are provided in Figure 1. The figure illustrates that the seasonality in the daily average temperature movements is

[1] Information from Government of Heilongjiang Province, 2017. http://www.hlj.gov.cn/sq/dldm/ [accessed April 19th , 2017]

similar to a sine function. The mean, maximum, minimum and standard deviations of historical daily average temperatures are provided in Table 1.

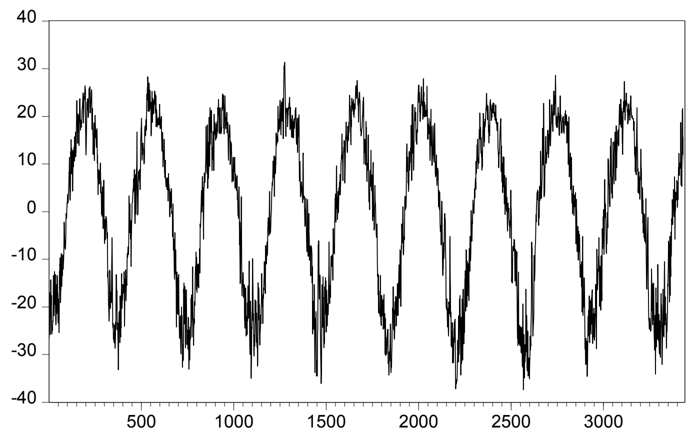

Figure 1: Daily Average Temperatures, 2006 through 2015, Heilongjiang

Table 1: Statistics of Daily Temperatures from years 1985 to 2015

Item	Obs.	Mean(°C)	Max.(°C)	Min.(°C)	S. D. (°C)
Temp	11322	-0.1650	31.30	-39.80	17.1162

Growing degree days (GDDs) are a measure of the heat to which crops are exposed during the growing season. Researchers found that GDD has high-order nonlinear effects on crop yields; yields are negatively impacted when growing season GDDs are too low or high (Sun & van Kooten 2014; Schlenker & Roberts 2008). GDDs in year *t* are calculated by subtracting 10°C from the average temperature for each day *d* in the growing-season, defined as: $GDD = \sum_{d=1}^{D} \mathrm{Max}(0, T_d - 10)$, where D (=153) refers to the number of days in the growing season (May to September) and T_d is the average temperature on day *d* (Sun & van Kooten 2014). The mean, maximum, minimum and standard deviations of historical GDDs are provided in Table 2.

Table 2: Statistics of GDDs from years 1985 to 2015

Item	Obs.	Mean(^{o}C)	Max.(^{o}C)	Min.(^{o}C)	S. D. (^{o}C)
Temp	31	1039.55	1212.00	871.60	92.9695

3 Methods

3.1 The Econometric Model

The underlying GDD weather index for the pricing of weather derivatives can be estimated using three methods: historical averages, econometric modeling and stochastic modeling. Among these methods, the historical averages is used to estimate GDD directly; however, the econometric model and stochastic models are constructed to estimate daily average temperatures, which are then used to compute the GDD index. Therefore, in this section, the daily average temperature is modelled by a econometric model with a sine function and a stochastic model with different mean-reverting speeds.

As is shown in the figure of daily average temperatures (Fig 1), temperatures clearly fluctuate in a manner approximating a sine function. As specified in previous research (Alaton et al. 2002), the following functional form for daily average temperature is assumed:

[1] $$T_t = b_0 + b_1 t + b_2 \sin(\omega t + \theta) + \varepsilon_t,$$

[2] $$\hat{T}_t = \hat{b}_0 + \hat{b}_1 t + \hat{b}_2 \sin(\omega t + \theta),$$

where $T_t = \frac{1}{2}(T_t^{\max} + T_t^{\min})$ is the mean of the daily maximum and minimum temperatures at day t (= 1, 2, …, 365), with one day omitted for leap years. $\hat{T}_t$ is the estimated deterministernistic sine component of temperature. Thus, while seasonal daily average temperatures ($\hat{T}_t$) follow a sine curve, the realized average temperature (T_t) on a given day t fluctuates randomly about that seasonal average. Further, since the oscillation period is one year, ω can be calculated by $2\pi/365$. As the yearly minimum and maximum temperatures do not usually occur at the troughs and peaks of a curve of a sine function, a phase angle θ is introduced in the sine function model. In addition, because the global temperature may get warmer as a result of climate change, there might be a positive upward trend in the data. b_i and θ are parameters to be estimated and t is a trend variable causing $\hat{T}_t$ to rise over time. For simiplicity of estimation, we can then rewrite equation [2] by trigonometric function as:

[3] $$\hat{T}_t = b_0 + b_1 t + b_2(\cos\theta \times \sin\omega \cdot t + \sin\theta \times \cos\omega \cdot t)$$

Therefore, the econometric model can be specified as:

[4] $$\hat{T}_t = b_0 + b_1 t + a_2 \sin\omega \cdot t + a_3 \cos\omega \cdot t$$

where b_0, b_1, a_2 $(= b_2 \cos\ \theta\ )$ and a_3 $(=\ b_2 \sin\ \theta\ )$ are parameters to be estimated.

3.2 Stochastic Model and Parameter Estimation

Based on estimated seasonal daily average temperatures, the daily average temperature could also be modeled by a stochastic process (Alaton et al., 2002). As temperatures cannot rise or fall indefinitely, a stochastic process model should not allow temperature to deviate much from its seasonal average in the long run. In other words, the stochastic process describing the temperature should have a mean-reverting property. Temperature can be modelled by the following mean-reverting process, which is an example of an Ito Process (Dixit & Pindyk 1994; Alaton et al. 2002):

[5] $$dT_t = \alpha(\hat{T}_t - T_t)dt + \sigma_t dw_t$$

where $\alpha\ (\hat{T}_t - T_t)$ is a drift term and $\sigma_t dw_t$ is the dispersion of the Weiner process w_t (Brownian motion), with $dw_t \sim N(0,\ \sqrt{dt})$ and σ_t is the volatility of the daily average temperature. In this case, T_t is the actual daily average temperature, $\hat{T}_t$ is the estimated daily average temperature for day t, and α is the speed of reversion to the estimated temperature. Thus, the stochastic difference equation [5] describes an Ornstein-Uhlenbeck process.

Assuming the start day is s and the final day is t, the solution to equation [5] is:

[6] $$T_t = \hat{T}_t + (T_s - \hat{T}_s)e^{-\alpha(t-s)} + \int_s^t e^{-\alpha(t-\tau)}\sigma_\tau dw_\tau$$

where $\tau \in [s, t]$ and other terms are defined as previously. In equation [6], $\hat{T}_t$ is the daily average temperature at time t as estimated by the econometric model. T_t and T_s are the estimated daily average temperatures for day t and s as derived by the stochastic process. σ_τ is the temperature variation for day t, which is assumed to be identical within a month, while the temperature variation $\hat{\sigma}_t$ can be represented by the standard deviations of each month. dw_τ represents the Weiner process or Brownian motion; α is the mean-reversion speed, which will be estimated using three different methods.

As Δt becomes infinitesimally small, the increment of a continuous Weiner process dw at time t can be represented as (Dixit & Pindyk 1994; Alaton et al. 2002):

[7] $$dw = \gamma_t (dt)^{1/2}$$

where $\gamma_t \sim N\ (0,\ 1\ )$ is a random variable that is serially uncorrelated.

The speed at which the process reverts back to the seasonal average is an important parameter(α). Three methods are used to estimate the parameter: (1) a martingale estimation function, (2) a first-order autoregressive process AR(1), and (3) a discrete-time data equation.

First, the martingale estimation function can also be used to estimate α. Based on Bibby and Sørensen (1995), Alaton et al. (2002) derive the following estimate of the mean-reversion parameter:

[8] $$\hat{\alpha} = -\ln\left|\frac{\sum_{t=1}^{N}\frac{\left(\hat{T}_{t-1}-T_{t-1}\right)\left(T_t-\hat{T}_t\right)}{\sigma_{t-1}^2}}{\sum_{t=1}^{N}\frac{\left(\hat{T}_{t-1}-T_{t-1}\right)\left(T_{t-1}-\hat{T}_{t-1}\right)}{\sigma_{t-1}^2}}\right|$$

where $\hat{T}_t$ is the estimated daily average temperature from the previous sine-function [4], T_t is again the realized average temperature, and σ_t is the mean variation of the realized daily average temperatures for day t.

Consider the AR(1) process for temperature:

[9] $$T_t = c_0 + c_1 t + c_2 \sin\ \omega \cdot t + c_3 \cos\ \omega \cdot t + c_{m1} AR_{\ t-1} + \ldots c_{mp} AR_{\ t-p} + c_{n1} MA_{\ t-1} \ldots c_{nq} MA_{\ t-q} + \delta_t$$

where c_i is parameters to be estimated, and δ_t is a error term. The estimated parameter c_{m1}, which measures the speed that today's temperature reverts back to yesterday's temperature, is identically the mean-reverting parameter α, so $\hat{\alpha} = c_{m1}$.

The parameters of the mean-reverting process could also be estimated using the discrete-time data equation (Dixit & Pindyk 1994):

$$[10] \quad T_t - T_{t-1} = d_0 + d_1 T_{t-1} + \zeta_t$$

where d_0 and d_1 are parameters to be estimated, and ζ_t is an error term. Then, by estimating the parameters in [11], we obtain a third estimate of the mean-reversion parameter as $\hat{\alpha} = -\ln(1 - \hat{d}_1)$.

3.3 Pricing Payoffs and Premiums of Put and Call Options

The premium of a contract is priced based on the expected payoff plus some precentage of payoff as profit. From the standpoint of the buyers, the payoff functions for put and call contracts are given by (Jewson et al. 2005):

$$[11] \quad p(x)_{put} = \begin{cases} D(K_1 - x), x \le K_1 \\ 0, x > K_1 \end{cases}$$

$$[12] \quad p(x)_{call} = \begin{cases} 0, x < K_2 \\ D(x - K_2), x \ge K_2 \end{cases}$$

where $p(x)$ is the payoff for an option; D is the tick size (dollar value per unit of the weather index); K_1 and K_2 are the strike (trigger) values for the put and call options, respectively; and x is the weather index. For put and call contracts, these are the payoffs against low and high values of the weather index, respectively.

Assuming that the weather index employed for a financial instrument follows a normal distribution, the expected payoff is (Jewson et al. 2005):

$$[13] \quad E_p = \int_a^b f(x) p(x) dx,$$

where $f(x)$ is the probability density function (PDF) of a weather index, which is GDD in the current study, and $p(x)$ is the payoff associated with the financial instrument for potential outcome x of the weather index. Denote the payoffs for put and call options as $p(x)_{put}$ and $p(x)_{call}$, respectively. Upon transforming the weather index into a standard normal distribution, let $z = \frac{x-\mu}{\sigma}$, the expected payoff function becomes:

[14] $$E_p = \int_{a'}^{b'} \phi(z)p(z)dz = \frac{1}{\sigma}\int_a^b \phi(z)p(x)dx,$$

where σ is the standard deviation of the weather index and $\phi(z)$ is the PDF of a standard normal distribution.

Inserting payoff functions [11] and [12] for the put and call contracts into the expected payoff function [14] gives the following respective closed-form functions for uncapped put and call options:

[15] $$E_{p,PUT} = \frac{1}{\sigma}\int_{-\infty}^{K_1} D(K_1 - x)\phi\left(\frac{x-\mu}{\sigma}\right)dx = D\sigma\phi\left(\frac{K_1-\mu}{\sigma}\right) + D\Phi\left(\frac{K_1-\mu}{\sigma}\right)(K_1-\mu),$$

[16] $$E_{p,CALL} = \frac{1}{\sigma}\int_{K_2}^{\infty} D(x - K_2)\phi\left(\frac{x-\mu}{\sigma}\right)dx = D\sigma\phi\left(\frac{K_2-\mu}{\sigma}\right) + D(\mu - K_2)\left[1-\Phi\left(\frac{K_2-\mu}{\sigma}\right)\right],$$

where μ is the mean value of the weather index; K_1 and K_2 are the lower and upper strike values, respectively; ϕ and Φ refer to the standard normal PDF and the cumulative density function (CDF), respectively; and x is the weather index.

Let $k_1 = \frac{K_1-\mu}{\sigma} = -m$ and $k_2 = \frac{K_2-\mu}{\sigma} = m$, where $m = \{0.2, 0.4, \ldots, 2.0\}$. Then equations [15] and [16] can be written as:

[17] $E_{p,PUT} = D\,\sigma\,[\,\phi(-m) - m\,\Phi(-m)]$ and

[18] $E_{p,CALL} = D\,\sigma\,[\,\phi(m) - m + m\,\Phi(m)]$.

The price of an option (or its premium) is calculated from the expected payoff as (Alaton et al. 2002):

[19] $c = (1+R)e^{-r(T-S)}E_p$,

where c is the premium that the hedgers (buyers) need to pay for a contract, r is a risk-free periodic market interest rate, S is the date that the contract is issued (purchased), and T is the date the contract is claimed or the expiration date. E_p is the expected payoffs based on the estimated value of GDDs. The seller of the option would expect a reward for taking on risk and, hence, the premium would be higher than the expected payoff by an amount known as the risk loading, which is often between 20% and 30% of the payoffs (Jewson et al. 2005). In the current application, we set the risk loading at 20%(R) of the expected payoff of the contract.

4. Results

4.1 Estimation Results of the Econometric Model

The econometric model is estimated by the method of GLS ARMA, with the regression results provided in Table 3. The estimated equation for daily average temperatures (ignoring the ARMA errors) can be written as :

[20] $\hat{T}_t = -\,4.60429 \times \sin w{\cdot}t - 22.87415 \times \cos w{\cdot}t$.

This model explains 97.3% of the variation in daily average temperatures. The LM test results for serial correlation are found in Table 4. The p-values are geater than the 5% significance level, indicating there is no serial correlation in the error term. The Root

Mean Squared Error (MSE) and Mean Absolute Error (MAE) between the actual daily average temperatures and the forecasts for daily average temperature with ARMA terms and without ARMA terms are provided in Table 5. It shows the difference for the MSE or MAE is very small, less than 0.005, so the forecast without ARMA terms can be a good substitution for that with ARMA terms. Figure 2 also shows that the predicted daily average temperatures fit well with the actual values.

Table 3: Estimation Results

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SINWT	-4.604294	0.256626	-17.9417	0.0000
COSWT	-22.87415	0.255700	-89.4569	0.0000
AR(1)	0.609351	0.088840	6.8590	0.0000
AR(2)	0.785809	0.135882	5.7830	0.0000
AR(3)	-0.430726	0.053991	-7.9778	0.0000
MA(1)	0.264928	0.088397	2.9970	0.0027
MA(2)	-0.769970	0.063583	-12.1097	0.0000
MA(3)	-0.232248	0.020557	-11.2976	0.0000
R^2	0.9731			
Adjusted R^2	0.9730			
S.E. of regression	2.8109			

Notes: Dependent variable: daily average temperature; Method: ARMA Generalized Least Squares (Gauss-Newton); coefficient for the intercept is not significant.

Table 4: LM Test for Serial Correlation

Breusch-Godfrey Serial Correlation LM Test			
F-statistic	0.341281	Prob. F(1,11306)	0.5591
Obs×R^2	0.341542	Prob. Chi-Square(1)	0.5589

Notes: Null Hypothesis: No serial correlation.

Table 5: Root Mean Squared Error and Mean Absolute Error of the Forecast

	Forecast with ARMA	Forecast without ARMA	Differences
Root Mean Squared Error	4.580475	4.583003	-0.00253
Mean Absolute Error	3.593020	3.594925	-0.00190

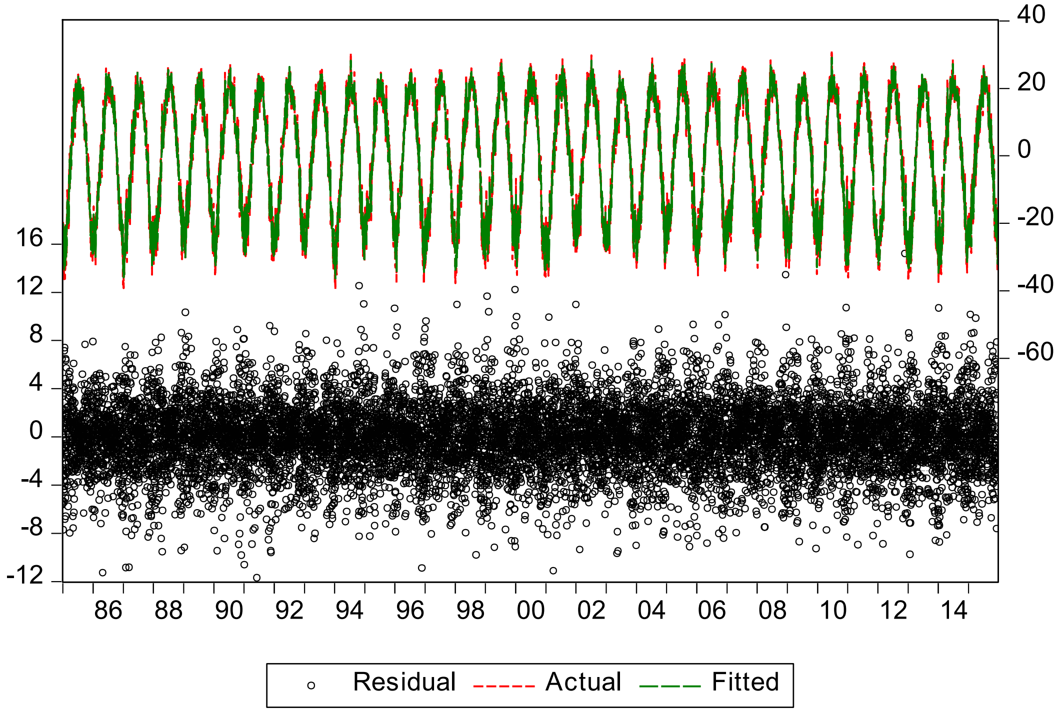

Figure 2: Residuals, Actual and Fitted daily average temperatures of the estimation

4.2 Estimation Results for Parameters in the Stotastic Process

From the 31-year historic daily average temperature data, the estimated values of σ for each month are given in Table 6. November and March have the largest standard deviations of 6.876 and 6.173, respectively, while July and August have the smallest, namely, 2.854 and 2.994, respectively. From the martingale estimation function [9], we find $\hat{\alpha}_1$=0.2547. From the the estimation results in Tables 3 and 7, the two alternative estimates of the mean reversion speed are $\hat{\alpha}_2$= 0.609351, which is estimated from the AR(1) process, and $\hat{\alpha}_3$=0.0161.

Table 6: Estimated σ and σ^2 for each of the 12 Months

Month	σ	σ^2
Jan.	5.6875	32.3477
Feb.	5.3814	28.9595
Mar.	6.1725	38.0998
Apr.	4.7146	22.2275
May.	4.4178	19.5170
Jun.	3.5336	12.4863
Jul.	2.8542	8.1465
Aug.	2.9938	8.9628
Sep.	4.2008	17.6467
Oct.	5.0146	25.1462
Nov.	6.8755	47.2725
Dec.	5.5608	30.9225

Table 7: Estimation Results for the discrete mean reversion speed (implied by equation 11)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TMEAN(-1)	-0.015977	0.001115	-14.33354	0.0000
R^2	0.008024			
Adjusted R^2	0.008024			
S.E. of regression	3.040854			

Notes: -ln(1-0.015977)= 0.01610601.

Fluctuations of historic and estimated (predicted) daily average temperatures are provided in Figure 2; using the sine function, these estimates fit the trend of the actual daily average temperatures quite closely. By adding a Wiener process to the sine function, the daily average temperatures are simulated with different mean reversion speeds – parameters from the Martingale estimation function, the AR (1) process and discrete-time data equation. These simulated daily average temperatures are then compared with the predicted values by the sine function without the stochastic process, with Root Mean Squared Errors plotted in Table 8. Finally, the weather index (GDD) is generated from the estimated daily average temperatures.

To compare the estimated GDDs by different methods and with different mean reversion parameters, the variations between the estimated and actual GDDs over the period 1990 through 2015 are plotted in Figure 4, with values presented in Table 9. The mean squared errors between estimated and actual GDDs, measured from the smallest to the largest, are those based on the econometric sine-function model, historical average method, and the stochastic process with a high mean reversion speed derived from the AR(1) process. The remaining estimated variations, with mean reversion speed from the martingale process and from the discrete time function, are much larger and, thus, are excluded from further analysis. In other words, the sine function without a stochastic process fit the actual GDDs best, followed by the historical average values of the GDDs, and then estimated GDDs from the stochastic process with a high mean reversion speed (α_1). Therefore, when pricing weather derivatives, the first choice is the the sine function without a stochastic process, then the weather index based on the historic means, and finally the GDD index derived using the mean reversion method with a high mean reversion parameter estimated from the AR(1) process.

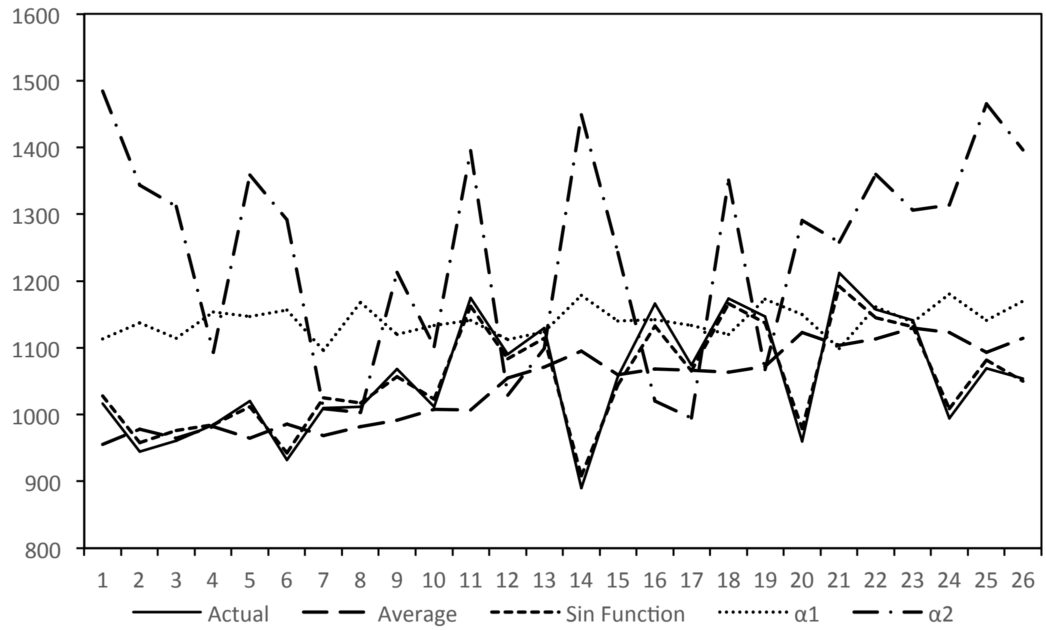

Figure 4: Estimated GDDs and actual GDDs, 1990-2015 (Actual: the actual GDDs; Average: the estimated GDDs from thehistorical average; Sine Function:the estimated GDDs from the econometric model; α_1: the estimated GDDs from the stochastic process with mean reversion speed α_1; α_2: the estimated GDDs from the stochastic process with mean reversion speed α_2)

Table 8: Variations between estimated and realized daily average temperatures, 1990-2015[a]

	Root Mean Squared Error
Forecast with ARMA	4.580475
Forecast Ignored ARMA	4.583003
α_1=0.609351	4.805958
α_2=0.2547	6.732413
α_3=0.016106	19.552905

Table 9: Variations between estimated and realized GDDs, 1990-2015[a]

Variation	Average	Standard Deviation	Root Mean Squared Error
Method 1: 5-year average	1043.77	59.6180	*83.8892*
Method 2: Sine function	1054.89	75.7210	*13.6631*
Method 3:Stochastic			
Mean reversion α_1	1068.62	23.5290	*133.8873*
Mean reversion α_2	1240.37	158.5006	263.7036
Mean reversion α_3	1141.37	1274.0740	1261.6840

[a] The average and standard deviation of the real GDDs are 1055.74 ^{o}C and 86.8813 ^{o}C; the actual standard deviation is used to calculate the prenimums for the contracts.

To price the financial weather derivatives, assume a tick size of D=\$1, a risk free interest rate r=0.08, Δt=0.75 year (time between the issue date and the expiry date), and risk loading of b=20%. Pricing results for the study region in northeastern China for 2015 are provided in Table 10. The actual GDDs for 2015 are 1053.70 ^{o}C; the actual standard diviation for years from 1990 to 2015 (=86.88 ^{o}C) is used to calculate the actual premiums for the contracts. The strike values are calculated by μ–0.2σ and μ+0.2σ for levels that are 0.2 standard deviations below and above of average realized GDDs, or m = –0.2 and m = 0.2; when m = –0.2 and 0.2, its probability densities and cumulative probabilities are given by: ϕ (–0.2)=0.3910, Φ(–0.2)=0.4207, ϕ (0.2)=0.3910 and Φ(0.2)=0.5792. The premiums are calculated from payoff equations [17] and [18] using [19], where R is a 20% risk loading factor. The premiums are determined for put and call contracts for year 2015 in which the strike level is above or below 0.2 standard deviations from the estimated weather index for the Average, Sine Function and α_1 approaches to estimating GDDs. In the following table, for the respective three approaches, the strike levels for put options are 1031.85 ^{o}C, 1039.75 ^{o}C and 1063.91 ^{o}C; the strike values for call options are 1055.70 ^{o}C, 1070.03 ^{o}C and 1073.32 ^{o}C. The prenimus of the put options of the average, sine function and α_1 stochastic approaches are $20.26, 25.74 and $8.00, respectively. The premium of the put option for 2015 based on the actual standard deviation over the period 1990 to 2015 is $29.53. It is shown that premiums from the econometric model with the sine function and historical average approaches are closer to those based on the realized weather values than the stochastic approach. Therefore, the econometric model with sine function and the historical average approaches provide better pricing estimates than other methods.

Table 10: Specification of GDD options for year 2015

<table>
<tr><th>Items</th><th>Put Option</th><th>Call Option</th></tr>
<tr><td>Weather Index</td><td>GDD</td><td>GDD</td></tr>
<tr><td rowspan="3">Strike Level
(K_1 or K_2)</td><td>Average:
1031.8504
(1043.774^{o}C–0.2×59.6179^{o}C)</td><td>Average:
1055.6976
(1043.774^{o}C+0.2×59.6179^{o}C)</td></tr>
<tr><td>Sine Function:
1039.7458
(1054.89^{o}C–0.2×75.7210^{o}C)</td><td>Sine Function:
1070.0342
(1054.89^{o}C+0.2×75.7210^{o}C)</td></tr>
<tr><td>α_1:
1063.9122
(1068.618^{o}C –0.2×23.5289^{o}C)</td><td>α_1:
1073.3238
(1068.618^{o}C +0.2×23.5289^{o}C)</td></tr>
<tr><td>Tick Size (D)</td><td>$ 1</td><td>$ 1</td></tr>
<tr><td>Premium[a]</td><td>$ 20.2654
$ 25.7392
$7.9980</td><td>$20.1452
$25.5865
$7.9506</td></tr>
<tr><td>Payoff</td><td>Max (K_1–GDD, 0)</td><td>Max (GDD–K_2, 0)</td></tr>
<tr><td>Issue date</td><td>December 31, 2014</td><td>December 31, 2014</td></tr>
<tr><td>Maturity date</td><td>September 31, 2015</td><td>September 31, 2015</td></tr>
</table>

[a] The premiums based on the actual GDDs (1053.70 ^{o}C) and its standard deviation (86.88 ^{o}C) for year 2015 is $29.5323 and $29.3571, with strike levels at 1036.32^{o}C and 1071.08^{o}C.

5. Conclusions

The agricultural sector is particularly vulnerable to weather risks, but financial weather derivatives can be developed to reduce farmers' exposure to such risk. This may particularly be the case for developing countries where a large portion of the population is still dependent on agriculture and government insurance and other support is lagging. Indeed, studies have shown that farmers in central and northwestern China, for example, are interested in weather indexed insurance (Turvey et al. 2009; Liu et al. 2010). Given that farmers are interested in financial weather products in China, the focus of this study is on the setting of premiums for puts and calls on growing-degree-day weather options.

Several methods were considered for forecasting future temperatures upon which to generate a GDD weather index. These in turn would determine the premiums that markets would charge, excluding transaction costs. In partcular, an econometric sine-function model and a stochastic model were compared with the historical average method. It is found that the econometric model without a stochastic process led to the best approximation of realized temperatures and that premiums for options based on a GDD weather index derived from the estimated econometric model were closer to the actual premiums than those derived using other methods. Further, if temperature was assumed to follow a stochastic process, the mean reversion parameter obtained from the AR(1) method gave a better result compared with other methods for mean reversion speed estimation.

Projecting future temperatures and growing degree days is fraught with uncertainty, which is why farmers wish to hedge against weather risk. However, markets need to provide farmers with hedges that are attractive, effective and truly representative of the risks producers encounter. Further research is required to better link crop yields to growing degree days – to match crop losses due to weather risks to the weather index – and to identify a proper tick size for pricing GDD-based weather derivatives.

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